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# Mathematical Structure of Two-Parameter Deformed Multimode Quantum Group $SL_{qs}(3)$

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The commutative relations of the generators of the two-parameter deformed multimode quantum group $SL_{qs}(3)$ are given, and irreducible $qs$-tensor operators of rank 1/2 of the multi-mode quantum group $SL_{qs}(2)$ are constructed.

Quantum groups (Drinfeld, 1985) have been discussed by many authors in the mathematics and physics literature. The general relations of quantum group $SL_q(3)$ were first given by Jimbo (1985). In the present paper we study the mathematical structure of the two-parameter deformed multimode quantum group $SL_{qs}(3)$

We first introduce three independent two-parameter deformed $k$-mode bosonic operators (Yu and Liu, 1998):

$$A_k = a_1 a_2 \ldots a_k \left\{ \frac{[n_1^a]_{qs}[n_2^a]_{qs} \ldots [n_k^a]_{qs}}{\min([n_1^a]_{qs}, [n_2^a]_{qs}, \ldots, [n_k^a]_{qs})} \right\}^{-1/2} \tag{1}$$

$$B_k = b_1 b_2 \ldots b_k \left\{ \frac{[n_1^b]_{qs^{-1}}[n_2^b]_{qs^{-1}} \ldots [n_k^b]_{qs^{-1}}}{\min([n_1^b]_{qs^{-1}}, [n_2^b]_{qs^{-1}}, \ldots, [n_k^b]_{qs^{-1}})} \right\}^{-1/2} \tag{2}$$

$$C_k = c_1 c_2 \ldots c_k \left\{ \frac{[n_1^c]_{qs}[n_2^c]_{qs} \ldots [n_k^c]_{qs}}{\min([n_1^c]_{qs}, [n_2^c]_{qs}, \ldots, [n_k^c]_{qs})} \right\}^{-1/2} \tag{3}$$

where the deformation brackets are defined as

$$[x]_{qs} = s^{1-x}[x] = s^{1-x}(q^x - q^{-x})/(q - q^{-1}), \qquad [x]_{qs^{-1}} = s^{x-1}[x] \tag{4}$$

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It is easy to check the following results:

$$A_kA_k^+ - s^{-1}qA_k^+A_k = (sq)^{-N_k^a}, \qquad A_kA_k^+ - (sq)^{-1}A_k^+A_k = (s^{-1}q)^{N_k^a} \quad (5)$$

$$[N_k^a, A_k^+] = A_k^+, \qquad [N_k^a, A_k] = -A_k \quad (6)$$

$$B_kB_k^+ - sq\, B_k^+B_k = (sq^{-1})^{N_k^b} \quad (7)$$

$$[N_k^b, B_k^+] = B_k^+, \qquad [N_k^b, B_k] = -B_k \quad (8)$$

$$C_kC_k^+ - s^{-1}qC_k^+C_k = (sq)^{-N_k^c}, \qquad C_kC_k^+ - (sq)^{-1}C_k^+C_k = (s^{-1}q)^{N_k^c} \quad (9)$$

$$[N_k^c, C_k^+] = C_k^+, \qquad [N_k^c, C_k] = -C_k \quad (10)$$

where

$$N_k^a = \min(n_1^a, n_2^a, \ldots, n_k^a) \quad (11)$$

$$N_k^b = \min(n_1^b, n_2^b, \ldots, n_k^b) \quad (12)$$

$$N_k^c = \min(n_1^c, n_2^c, \ldots, n_k^c) \quad (13)$$

Similar to the quantum group $SL_q(3)$ (Jimbo, 1985), we choose the Chevalley base:

$$h_1 = N_k^a - N_k^b, \qquad h_2 = N_k^b - N_k^c \quad (14)$$

$$e_1 = A_k^+B_k \quad, \qquad e_{-1} = B_k^+A_k \quad (15)$$

$$e_2 = B_k^+C_k \quad, \qquad e_{-2} = C_k^+B_k \quad (16)$$

which obey the relations

$$[h_i, e_{\pm i}] = \pm 2e_{\pm i} \qquad (i = 1, 2) \quad (17)$$

$$[h_i, e_{\pm j}] = \mp e_{\pm j} \qquad (i \neq j, \quad i, j = 1, 2) \quad (18)$$

$$e_1e_{-1} - s^2e_{-1}e_1 = [h_1]_{qs} \quad (19)$$

$$e_2e_{-2} - s^{-2}e_{-2}e_2 = [h_2]_{qs^{-1}} \quad (20)$$

and Serre's relations

$$e_1^2e_2 + s^2e_2e_1^2 = [2]_{qs^{-1}}e_1e_2e_1 \quad (21)$$

$$s^2e_{-1}^2e_{-2} + e_{-2}e_{-1}^2 = [2]_{qs^{-1}}e_{-1}e_{-2}e_{-1} \quad (22)$$

In this group, two additional operators are defined as

$$e_3 = s^{N_k^c}A_k^+C_k, \qquad e_{-3} = C_k^+A_ks^{N_k^c} \quad (23)$$

which satisfy the commutative relations

$$[h_i, e_{\pm 3}] = \pm e_{\pm 3} \qquad (i = 1, 2) \tag{24}$$

$$e_3 e_{-3} - s^2 e_{-3} e_3 = [h_1 + h_2]_{qs} \tag{25}$$

and Serre's relations

$$s^2 e_{-1}^2 e_3 + e_3 e_{-1}^2 = [2]_{qs^{-1}} e_{-1} e_3 e_{-1} \tag{26}$$

$$e_1^2 e_{-3} + s^2 e_{-3} e_1^2 = [2]_{qs^{-1}} e_1 e_{-3} e_1 \tag{27}$$

We now redefine the generators of the two-parameter deformed *k*-mode quantum group $SL_{qs}(3)$ as follows:

$$J_0 = \tfrac{1}{2} h_1, \qquad J_\pm = e_{\pm 1}, \qquad Q = -(h_1 + 2h_2) \tag{28}$$

and

$$T_{1/2} = -e_{-2}, \qquad T_{-1/2} = e_{-3}, \qquad V_{-1/2} = e_2, \qquad V_{1/2} = e_3 \tag{29}$$

Obviously, we have

$$V_t = (-1)^{1/2-t} (T_{-t})^+ \qquad (t = \pm \tfrac{1}{2}) \tag{30}$$

It is easy to prove the following results:

$$[Q, J_0] = [Q, J_\pm] = 0 \tag{31}$$

$$[J_0, J_\pm] = \pm J_\pm, \qquad s^{-1} J_+ J_- - s J_- J_+ = s^{-2J_0} [2J_0] \tag{32}$$

$$[J_0, T_t] = t\, T_t, \qquad [J_0, V_t] = t V_t \qquad (t = \pm \tfrac{1}{2}) \tag{33}$$

$$[Q, T_t] = 3\, T_t, \qquad [Q, V_t] = -3 V_t \qquad (t = \pm \tfrac{1}{2}) \tag{34}$$

and

$$s^2 J_-^2 T_{1/2} + T_{1/2} J_-^2 = [2]_{qs^{-1}} J_- T_{1/2} J_- \tag{35}$$

$$J_+^2 T_{-1/2} + s^2 T_{-1/2} J_+^2 = [2]_{qs^{-1}} J_+ T_{-1/2} J_+ \tag{36}$$

In particular, as $s \to 1$, the above results reduce to the case of the two-parameter deformed *k*-mode quantum group $SU_q(3)$. On the other hand, we see that the operators $T_t$ and $V_t$ can be as 1/2-rank tensors of the two-parameter deformed *k*-mode quantum group $SL_{qs}(2)$.

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